arXiv:1201.1590v1 [gr-qc] 7 Jan 2012

Pseudo-topological transitions in 2D gravity models coupled to massless scalar fields

J. Ambjørn [a,c], *A. T. Görlich* [a,b,d], *J. Jurkiewicz* [b,e] and *H.-G. Zhang* [b,f]

[a] The Niels Bohr Institute, Copenhagen University
Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark.

[b] Mark Kac Complex Systems Research Centre,
Marian Smoluchowski Institute of Physics, Jagellonian University,
Reymonta 4, PL 30-059 Krakow, Poland.

Abstract

We study the geometries generated by two-dimensional causal dynamical triangulations (CDT) coupled to d massless scalar fields. Using methods similar to those used to study four-dimensional CDT we show that there exists a $c = 1$ "barrier", analogous to the $c = 1$ barrier encountered in non-critical string theory, only the CDT transition is easier to be detected numerically. For $d \leq 1$ we observe time-translation invariance and geometries entirely governed by quantum fluctuations around the uniform toroidal topology put in by hand. For $d > 1$ the effective average geometry is no longer toroidal but "semiclassical" and spherical with Hausdorff dimension $d_H = 3$. In the $d > 1$ sector we study the time dependence of the semiclassical spatial volume distribution and show that the observed behavior is described an effective mini-superspace action analogous to the actions found in the de Sitter phase of three- and four-dimensional pure CDT simulations and in the three-dimensional CDT-like Hořava-Lifshitz models.

[c] email: ambjorn@nbi.dk
[d] email: goerlich@nbi.dk, atg@th.if.uj.edu.pl
[e] email: jerzy.jurkiewicz@uj.edu.pl
[f] email: zhang@th.if.uj.edu.pl

1 Introduction

The formalism of dynamical triangulations (DT) [1] was introduced to provide a lattice regularization of Polyakov's theory of non-critical strings [2] and 2d quantum gravity, and later of both 3d and 4d quantum gravity [3]. It was successful in the 2d case, but no interesting continuum limit has so far been found in the pure gravity sector of the higher dimensional lattice quantum gravity theories [4]. The problem was that two unphysical sectors, the so-called "crumpled phase" where geometries have a Hausdorff dimension $d_H = \infty$, and the branched polymer (BP) phase, where the Hausdorff dimension of the geometries was $d_H = 2$, where separated by a first order phase transition[1].

Causal Dynamical triangulations (CDT) [6, 7, 8] were introduced to cure the problems encountered in the three- and four-dimensional Euclidean DT quantum gravity theories. The idea was to insist on a (proper time) foliation[2], which would curb the formation of some of the "pathological" DT configurations [9], and indeed some success has been obtained. A semiclassical regime of bare coupling constants has been located (the so-called de Sitter phase) [15], quantum universes of the size of 10-20 Planck units have been studied [16], and a second order phase transition line, where a continuum UV limit might exist, has been located [17].

Except for a few features of 3d CDT [18], all the studies of the higher dimensional CDT theories have used computer simulations and the precise relation of the CDT theories to the old DT theories is not clear[3]. In this respect two-dimensional CDT is different. This theory can be solved analytically and its relation to the two-dimensional Euclidean quantum gravity, as solved by the DT model, is also clear. The CDT theory can be viewed as an effective theory, where baby-universes created in the DT-theory have been integrated out [20]. As a result the Hausdorff dimension of the ensemble of 2d CDT geometries is equal to the canonical dimension, $d_H = 2$, while the Hausdorff dimension of the ensemble of DT geometries is four[4] [22, 23, 24, 25, 26].

One of the amazing features of the DT theory of two-dimensional quantum

[1]Attempts to obtain different Hausdorff dimensions by adding matter or using different weight factors for the path integral led to the possibility of a new phase, the "crinkled phase" [5], which seems to have Hausdorff dimension close to four.

[2]The idea of a time foliation as important part of a theory of quantum gravity has also been advocated recently by Hořava [10]. The Hořava-Lifshitz theory of gravity might be closely related to the CDT theory of gravity. One observes the same spectral dimension in the four-dimensional theories [11, 12], and the same kind of phase diagram [13]. Also in three dimensions there is a close connection [14].

[3]A recent attempt [19] to revive the so-called crinkled phase of DT may hint a connection. One observes a behavior of the spectral dimension which is almost identical to the one observed in CDT.

[4]It is possible to introduce an additional coupling constant related to the creation of baby universes and follow the detailed transition from CDT geometries to DT geometries [21].

gravity is that it can be solved analytically for any minimal rational conformal field theory coupled to geometry, as long as the central charge c of the conformal theory is less than or equal to 1, and one finds agreement with the corresponding continuum results from quantum Liouville theory [27]. For $c > 1$ the analytical solutions become unphysical (complex critical exponents) and it is believed that the strong interaction between matter and geometries forces the geometries to degenerate into branched polymers, an observation going all the way back to the study of random surfaces on hyper-cubic lattices and random surfaces using DT [28, 29].

While it is straightforward to couple matter to the 2d CDT theory [30, 31], one can unfortunately not (yet) solve the theory analytically. However, numerically one can still investigate the theory. One finds that for the Ising model and the three-state Potts model coupled to CDT, which have $c = 1/2$ and $c = 4/5$, the critical exponents of matter remain equal to the flat space exponents. This is in sharp contrast to the situation when using the 2d DT regularization, where the critical exponents change from their flat space values to their so-called KPZ values [27]. Since the ensemble of CDT geometries can be derived from the DT ensemble of geometries by integrating out baby universes, it is natural to associate the KPZ exponents with baby universe creation, an intuition which has been made quite concrete for the entropy exponent γ [32, 33, 34], and an observation made before the invention of the CDT formalism. Thus it is natural to conjecture that CDT coupled to conformal field theories with $c \leq 1$ can still be viewed as effective DT theories where baby universes have been integrated out, although no analytic proof exists presently. The interesting question which we want to address in this article is what happens in CDT when one crosses the $c = 1$ barrier. As mentioned above, the KPZ exponents become complex, signaling a breakdown of the concept of a two-dimensional surface on which the conformal matter lives into branched polymers. The formation of local spikes [35] when $c > 1$ is believed to be the explanation of this world sheet instability.

In the CDT case the geometry cannot degenerate into BP by construction. Is there a $c = 1$ barrier at all? In [30] a hint of a different behavior for $c > 1$ was seen: for 8 Ising spins (i.e. $c = 4$) coupled to CDT it was observed that the average geometry was very different from the $c < 1$ geometry. In the following we will provide evidence that the observation made in [30] is not accidental, that the geometry undergoes a phase transition for $c > 1$, and that this transition is easier to observe from a numerical point of view than the "old" DT transition. The transition is not to a BP phase, but to a phase of "spherical, semiclassical" geometry, which resembles somewhat the geometry encountered in higher dimensional CDT.

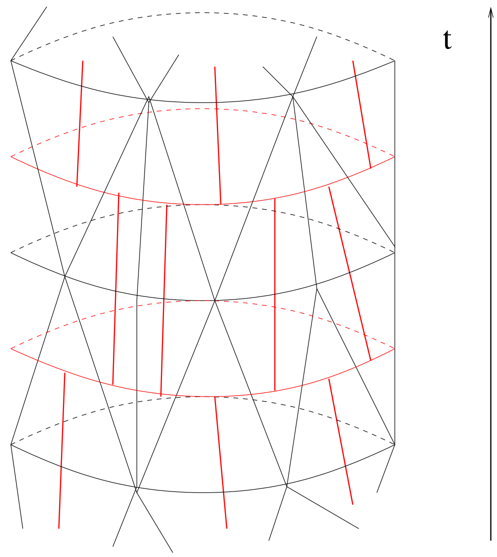

Figure 1: Direct and dual lattices of Causal Dynamical Triangulations in 2D.

2 Choice of matter and numerical setup

In order to study the effect of conformal matter on the CDT geometry we have chosen to use massless scalar fields rather than the Ising spins used in [30]. Using Ising spins is more demanding from a numerical point of view. The behavior of the Ising system is governed by the strength of the spin-spin coupling[5] β. For large β the system is magnetized and effectively decouples from the geometry. Similarly, for small β the spins flip uncorrelated, again largely independent of the geometry. At a certain critical $\beta = \beta_c$ there is a transition between the magnetized and demagnetized phase, and only at this point there is a significant coupling between geometry and matter, corresponding to a $c = 1/2$ conformal field theory coupled to geometry. However, we first have to locate β_c numerically, and contrary to the situation for a fixed geometry, β_c will depend on the number of Ising spins we use. Massless Gaussian fields are automatically critical and we have no such fine-tuning problems.

The geometric setup is the following: we want to have a time foliation of our 2d geometries with a given proper time T. We assume that time has been Wick-rotated such that our 2d space-time has Euclidean signature, and finally we assume that time is periodic. In the CDT discretization we thus discretize the proper time in T discrete units of length a, a being the link length of the *equilateral* triangles we will use. a will play no role in the sense that it can

[5]To be more precise, denoting the spin coupling J, we have $\beta = J/kT$, where T is the temperature and k Boltzmann's constant.

be absorbed in coupling constants with dimension and we choose $a = 1$ in the following. Thus the discretized time takes the values $t = 0, \dots, T-1$. The spatial slice at time t is assumed to have the topology of S^1 and in the CDT regularization it consists of $L(t)$ links. The space-time slab between t and $t+1$ will then be formed by a triangulation of $L(t)$ triangles which have two vertices at t and one vertex at $t+1$, and $L(t+1)$ triangles with two vertices at $t+1$ and one vertex at t, these $V(t) = L(t) + L(t+1)$ triangles glued together such that the topology of the slab is $S^1 \times [0,1]$. Since time is taken periodic the global topology of our 2d Euclidean space-time will be that of the torus, $T^2 = S^1 \times S^1$ and the total space-time volume is proportional to the total number of triangles

$$N = \sum_{t=0}^{T-1} V(t). \tag{1}$$

A natural way to couple matter to geometry in CDT is to place the matter fields at the center of the triangles. Since we are using equilateral triangles the Gaussian action-term associated with such matter assignment for a given triangulation $\mathcal{T}$is trivial:

$$S_{Gauss}(\mathcal{T}, x^\mu) = \sum_{\langle i,j \rangle} \sum_{\mu=1}^{d} (x_i^\mu - x_j^\mu)^2. \tag{2}$$

Here the summation is over d Gaussian fields, and over neighboring triangles i,j. A configuration in the path integral is characterized by a geometry, i.e. a triangulation $\mathcal{T}$, and a field configuration assigned to that geometry, i.e. $d \times N$ values x_i^μ. The probability amplitude, or (since we work with Euclidean signature) the Boltzmann weight, of the configuration is then

$$P(\mathcal{T}) \propto e^{-S(\mathcal{T})}, \quad S(\mathcal{T}) = S_{Gauss}(\mathcal{T}, x^\mu) + \Lambda N(\mathcal{T}) + \epsilon (N(\mathcal{T}) - \bar{N})^2. \tag{3}$$

Here Λ denotes a cosmological term and term $\epsilon(N - \bar{N})^2$ is added to insure that the space-time volume fluctuates around a prescribed value $\bar{N}$.

Finally the path integral (the state sum or partition function in the statistical mechanics interpretation) is formally

$$\mathcal{Z} = \sum_{\mathcal{T}} \frac{1}{C_T} \int \prod_{i,\mu} dx_i^\mu \; e^{-S(\mathcal{T})}, \tag{4}$$

where C_T is a symmetry factor of the graph $\mathcal{T}$, the order of the so-called automorphism group of $\mathcal{T}$. There is no need of a coupling constant in front of the Gaussian action (as already mentioned) since such a coupling can be viewed as a rescaling $c\, x_i \to y_i$ which in the path integral can be compensated by a change

of $\Lambda \to \Lambda' = \Lambda - d \log c$. The path integral (4) contains a zero mode of the field, namely the constant field, which has to be fixed in order for the integral to be well defined.

Finally, since we have placed the matter fields at the center of the triangles, it is natural to use the graph dual to the triangulation, i.e. a ϕ^3 graph. Triangles are then represented as vertices and what we in such a ϕ^3 graphs will call a spatial line, corresponds to half-integer times in the triangulation picture, being of length $V(t)$ (see Fig. 1). The dual graph, which we also denote $\mathcal{T}$, thus has $N(\mathcal{T})$ vertices i, and each vertex has two "space-like" links (horizontal links) and one "time-like" (vertical) link pointing either up or down. This description of CDT using dual graphs was first used in [36].

We can use Monte Carlo simulations to calculate the expectation values of certain "observables" in the statistical ensemble defined by eq. (4). In order to use Monte Carlo simulations we have to be able to move ergodically between the configurations. This means we must be able to change geometry and field configurations in such a way that successive changes can bring us to any configuration. If changes made on the configurations satisfy the so-called detailed balance condition successive updatings will under quite general conditions lead to the correct probability distribution, independent of the starting configuration [37].

The geometric update on the ϕ^3 graph is performed using two so-called moves: the move "add", which adds a new vertical link (and the two corresponding vertices) and the inverse move "del", where the vertical link (and the two corresponding vertices) is deleted. The two moves satisfy the detailed balance condition and the geometric constraints, which do not permit to break the system into disjoint parts. As already mentioned the scalar fields x_i^μ, $\mu = 1, \ldots d$ are localized at the vertices. They are assumed periodic both in space and time.

In numerical simulations the field values enter the detailed balance condition. Geometric moves are supplemented by the local update of the matter fields, performed after a sweep composed of N "add" and "del" moves, performed with the same a priori probability. The matter fields are updated using the heat bath algorithm. In the discussion below we shall be concentrated only on the distribution $V(t)$ as a function of a discrete time t. We shall integrate over all values of the fields. The distributions are independent of the zero modes of the scalar fields.

3 Numerical results

3.1 The semiclassical configurations for $d > 1$

If a class of semiclassical field configurations dominates the path integral, expanding around any such configuration will break translational invariance unless it is

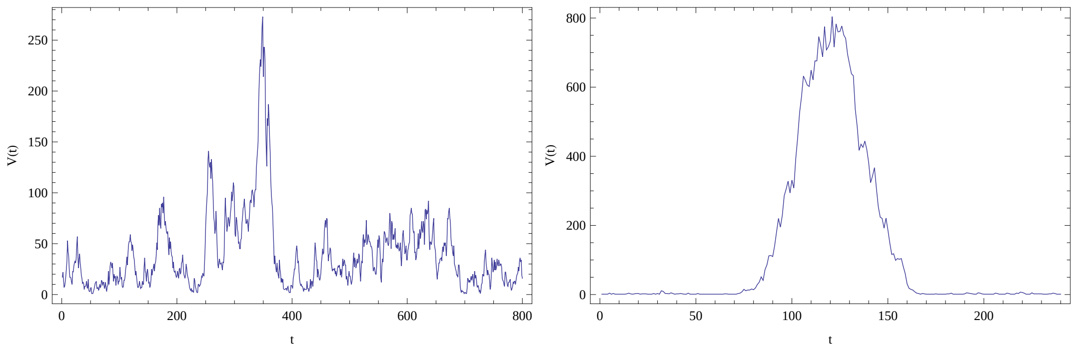

Figure 2: Examples of configurations with one and four scalar fields coupled to CDT geometry. The plots are for $N = 64000$ and respectively $T = 800$ and $T = 240$.

the constant field configuration. Usually the action is translationally invariant and the zero-modes associated with this invariance should be treated as collective coordinates which will restore the invariance of the partition function when expanded around semiclassical solutions. Rather surprisingly, we will encounter such a situation when $d > 1$.

When $d > 1$ and we just look at a typical geometric configuration, as it presents itself in the path integral, we observe a "blob" if we plot $V(t)$ as a function of the proper time t. By a "blob" we mean that almost all the spatial volume is located in a region of finite extension, say ΔT, which is independent of T if only T is sufficiently large. ΔT will depend on N and grow with N as described below, but for a fixed N ΔT is, up to fluctuations, the same for all *typical* configurations, i.e. configurations we pick randomly from the computer[6]. Outside an interval of size ΔT $V(t)$ is basically zero, only we do not allow, *by construction*, the universe to shrink to zero spatial size. Thus we have outside ΔT a *stalk* of cut-off size, superimposed with small fluctuations. To illustrate this behavior we show plots representing typical spatial volume distributions $V(t)$ versus time for $d = 1$ and $d = 4$ in Fig. 2. For $d = 1$ we do not observe the blob structure.

The "center of mass" of the blob will perform a random walk as a function of computer time, due to translational time invariance of the action. Thus an average of $V(t)$ will just result in a uniform distribution. Clearly this will not capture the true nature of the configurations for $d > 1$, and we have in fact to "undo" the collective mode integration mentioned above associated with an expansion around a semiclassical solution. For each configuration we thus fix the

[6]Of course *all* configurations are present in the path integral, also configurations which spread out over the whole range T. However, when N is large the probability they will be picked by a random selection will be very small.

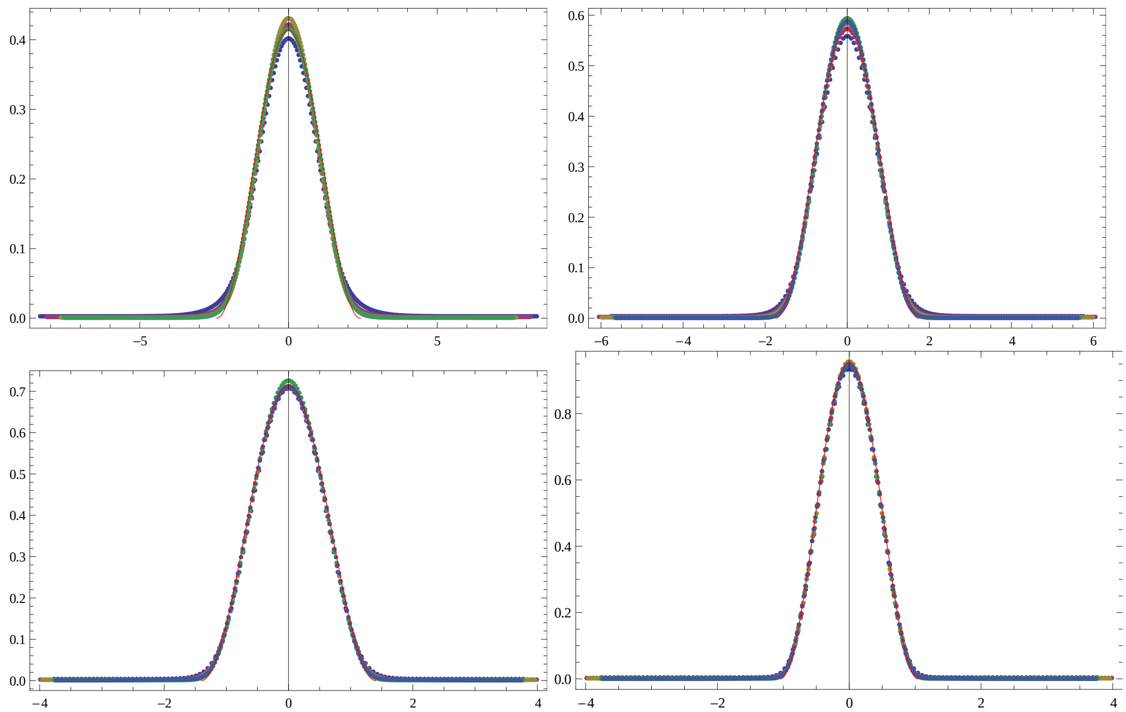

Figure 3: Universality of the rescaled volume distributions for various number of scalar fields. The top figures represent $d = 2$ and $d = 3$ for $N = 8000,\ 16000,\ 32000,\ 64000,\ 128000$. The time T used depended on volume and was respectively $T = 320,\ 400,\ 480,\ 600,\ 750,\ 1000$ for $d = 2$ and $T = 150,\ 180,\ 240,\ 300,\ 360,\ 450$ for $d = 3$. The bottom figures correspond $d = 4$ and $d = 6$ for the same range of spatial volumes. The time was in both cases $T = 100, 120,\ 160,\ 200,\ 240,\ 300$.

position of the "center of mass" of the spatial volume distribution to be at $t = 0$ and define time to be symmetric around $t = 0$. We then average the distribution over many statistically independent configurations, obtaining in this way what we call a "semiclassical distribution", in the sense that configurations with the "blob" spatial volume profiles are the dominant ones in the path integral.

For a given $d > 1$ we have studied the universal behavior as a function of N, the size of our system. We assume the blob (and only the blob, not the stalk) is characterized by a Hausdorff dimension d_H, measuring the space-time volume enclosed inside a geodesic[7] ball of radius r: $V(r) \propto r^{d_H}$. We then expect that the quantity

$$\rho(\tau) = V(t) N^{1/d_H - 1}, \tag{5}$$

plotted as a function of the scaled time variable $\tau = t/N^{1/d_H}$ should be a universal

[7]We use here as geodesic distance the link distance between two vertices on the ϕ^3 graph.

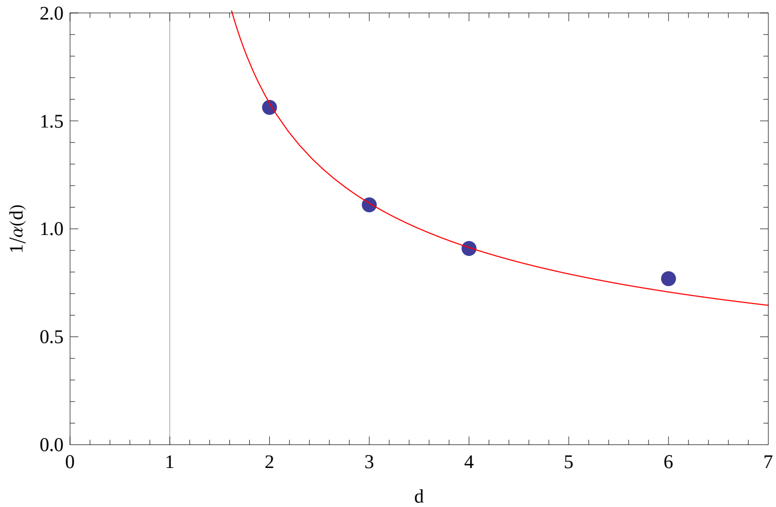

Figure 4: The dependence of $1/\alpha$ on d is shown. The line is const.$/\sqrt{d-1}$.

function, independent of the total volume N. Fig. 3 illustrates this scaling for $d = 6, 4, 3$ and 2 with $d_H = 3$. The continuous curve is a plot of

$$\rho_0(\tau) = \frac{2\alpha}{\pi} \cos^2(\alpha\tau) \tag{6}$$

with a coefficient α depending on d. The dependence of $1/\alpha(d)$ is presented in the Fig. 4, from which we find that α increases with d.

3.2 The effective action

It is an obvious question what kind of action creates the observed blob-structure? From the studies of 4d CDT we know how to extract the effective action [8, 16] for such a system. We want to get the effective action as a function of $V(t)$, and we can obtain the quadratic part by analyzing the covariance matrix

$$C_{t,t'} = \langle V(t)V(t')\rangle - \langle V(t)\rangle\langle V(t')\rangle \tag{7}$$

for a given d and N. As argued in [8, 16], inverting such a matrix gives an "inverse propagator" $P(t,t')$, or a matrix of second derivatives of $S_{eff}[V(t)]$ with respect to $V(t)$ and $V(t')$. We find the structure very similar to that in the 4D case, with non-zero matrix elements concentrated on the diagonal and the two off-diagonals. The observed behavior is consistent with the effective action

$$S_{eff} = \frac{1}{\Gamma} \sum_t \left(\frac{(V(t) - V(t+1))^2}{V(t) + V(t+1)} - \lambda V(t) \right). \tag{8}$$

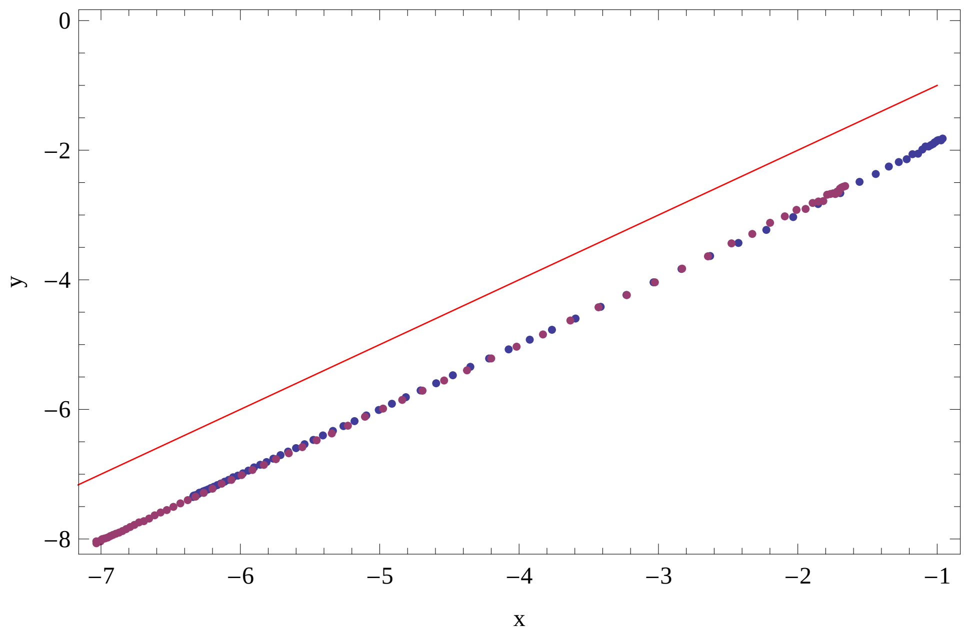

Figure 5: The diagonal and the first off-diagonal of the inverse matrix versus the theoretical values (see text).

In this equation λ should be viewed as a Lagrange multiplier which is determined by the condition that the integral of $V(t)$ is equal to $\bar{N}$, see eq. (1). The equation represents a naive discretization of the mini-superspace action in three dimensions, in accordance with our measurement that $d_H = 3$.

The figure 5 shows, on a log-log plot, the comparison of the measured values of the diagonal and the first off-diagonal matrix elements (multiplied by a factor (-2)) (the y axis) versus the same values calculated as derivatives of (8), using experimental values of $\langle V(t) \rangle$ (the x axis). The system is for $d = 4$, $N = 128000$ and $T = 300$. The straight line shown on the plot is $y = x$. The (constant) shift between the experimental plots and the straight line can be used to determine the value of Γ.

Applying the same method to extract the semi-classical volume distributions for $d = 0$ and $d = 1$ where we have "unbroken symmetry", i.e. no blob-formation, leads to configurations centered around the largest volume. As a consequence the average volume distribution is not flat (as we would expect for the toroidal geometry with no blob-formation), but has a maximum at zero time by construction. More specifically, since there is no dominant blob, but rather a a sequence of spatial volume fluctuations with varying length in time t, we pick for each configuration the largest fluctuation and measure distributions centered around this fluctuation. Clearly, the average distributions $\langle V(t) \rangle$ obtained this way for $d = 0$

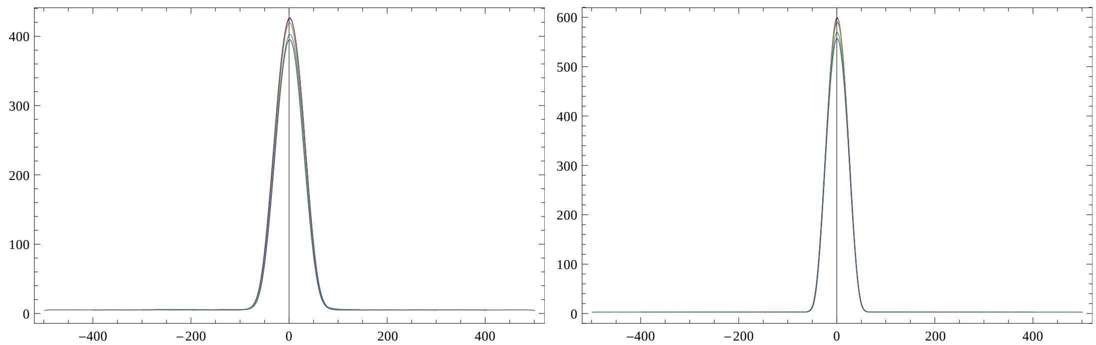

Figure 6: The dependence of the semi-classical volume distribution on T for two and three scalar fields, volume 64k. In both cases we use $T = 200,\ 240,\ 320,\ 400,\ 480,\ 600$.

and $d = 1$ are very different from the distributions observed for $d > 1$, having no stalk, but spreading out when T is increased and N kept fixed (see Fig. 7 and compare to Fig. 6 where $d > 1$ and we have blob-formation).

To summarize:

- For $d > 1$ the shape of the semi-classical distribution is practically independent of T (see the Fig 6). There is a small shift near the maximum, since in all plots we have the same volume.

- For $d \leq 1$ the width $w_T(d)$ grows linearly with T when the period T is increased and this effect is shown on the Fig. 7 for $d = 0$ (pure gravity) and for $d = 1$. The dependence of the width $w_T(d)$ on d and T is shown in Fig. 8.

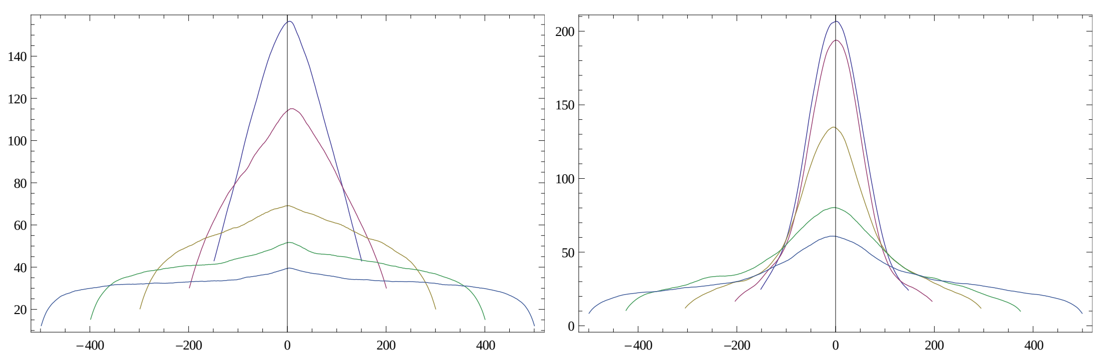

Figure 7: The dependence of the semi-classical volume distribution on T for pure gravity and one scalar field, volume 64k. In both cases we use $T = 200,\ 240,\ 320,\ 400,\ 480,\ 600$.

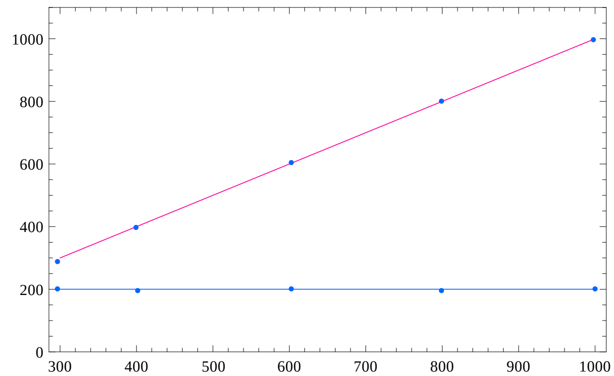

Figure 8: The relation between $w_T(d)$ on d with different scalar fields in fixed volume 64k. The red curve is pure gravity and the blue curve is $d = 2$. We use $T = 300,\ 400,\ 600,\ 800,\ 1000$ for both cases.

4 Discussion and conclusion

We have studied the causal dynamical triangulation (CDT) model for quantum gravity in two dimensions coupled to d massless scalar fields. The topology of the space-time was chosen to be $S^1 \times S^1$, i.e. both space and (Euclidean) time were chosen periodic.

Knowing that pure two-dimensional CDT can be viewed as an effective theory of pure ($c = 0$) Euclidean quantum gravity where baby universes have been integrated out, we asked if one can observe any trace of the $c = 1$ barrier known in Euclidean quantum gravity if we coupled conformal matter to CDT 2d gravity. Somewhat surprisingly the result is not only yes, but the numerical signal is much clearer than what is seen in the (lattice versions) of Euclidean quantum gravity [38, 39].

When $d > 1$ we observe what we have called "semiclassical" configurations which dominate the path integral. These are "universes" which have a spatial volume distribution $V(t)$ described by (6), i.e. a distribution much like one would have if the blob had the geometry of S^3, or an elongated version of S^3 as described in [40]. A corresponding mini-superspace action is given by (8). From the finite sized scaling relation (5) which is very well satisfied we obtain the Hausdorff dimension of the blob to be 3. This seems to be valid for all $d > 1$, but with the blob structure more pronounced with increasing d. It is also consistent with the results obtained in [30], where the system of 8 Ising spins coupled to 2d CDT was studied. Also here a blob with $d_H = 3$ was observed (although it was not checked if the distribution of $V(t)$ was given by (6)). Since this is a very different matter system, seemingly both the formation of a blob and its Hausdorff dimension are independent of the kind of conformal matter coupled to the CDT geometry, as

long as $c > 1$.

Since the effective action by construction reproduces the volume-volume correlator, this action will also describe (to quadratic approximation) the quantum fluctuations around the semiclassical blob.

The way we use the d Gaussian fields in the Monte Carlo simulations forces d to be a non-negative integer. Thus it is difficult to address the exact nature of the CDT $c = 1$ "barrier" since we start at $c = 2$. However, since the action is Gaussian it is in principle possible to integrate out the matter fields for each triangulation $\mathcal{T}$. Let $D_{\mathcal{T}}(i, j)$ denote the so-called coincidence matrix of the triangulation $\mathcal{T}$, an $N \times N$ matrix which is 3 in the diagonal and -1 if i and j are neighboring vertices in the ϕ^3-graph $\mathcal{T}$. Let $\Delta'(\mathcal{T})$ be the determinant of $D_{\mathcal{T}}(i, j)$ with the zero mode removed. We have

$$\int \prod_{i,\mu} d'x_i^\mu \; e^{-S_{Gauss}(\mathcal{T}, x^\mu)} \propto \Big(\Delta'(\mathcal{T})\Big)^{-d/2}, \tag{9}$$

where d' symbolizes that the integration excludes the translational zero-mode. Thus the partition function $\mathcal{Z}$ from eq. (4) can be written as

$$\mathcal{Z} = \sum_{\mathcal{T}} \frac{1}{C_T} \; e^{-\Lambda N(\mathcal{T} - \epsilon(N(\mathcal{T}) - \bar{N})^2)} \Big(\Delta'(\mathcal{T})\Big)^{-d/2}. \tag{10}$$

The advantage of this representation is that it is a continuous function of the number of Gaussian fields and we can thus use it to approach $d = 1$ from above. However, the disadvantage of the expression is that we have to calculate the $N \times N$ determinant when we update geometries. It can be done when N is less than a few hundred [41], but such N might be too small to signal clearly a phase transition.

The geometric structure observed here is similar to that observed in CDT formulation of the 3D and 4D quantum gravity. Also in this case the geometries which dominate the path integral are *not* (Euclidean) time-translational invariant configurations, and the systems in the so-called (Euclidean) de Sitter phase can be viewed as semi-classical 3D and 4D spheres with superimposed quantum fluctuations. Hopefully the fact that matter seems to trigger the same kind of configurations will help us understand in more detail the mechanism responsible for creating the regular spherical semi-classical geometry in higher dimensional CDT.

Clearly our "spheres" with Hausdorff dimension 3 cannot be three-dimensional spheres in an ordinary sense since they are constructed by triangles, i.e. two-dimensional building blocks. A measurement of the spectral dimension of the spheres would be very interesting, in particular since the spectral dimension in higher dimensional CDT seemingly has quite surprising characteristics linking it

both to Hořava-Lifshitz gravity and UV fix points in asymptotic safety scenarios [42].

Another interesting aspect to investigate is the effect of a mass term. If we add a mass to the Gaussian field, the field theory is no longer a conformal field theory. Naively, since we try to formulate a quantum theory of gravity, one would expect such a mass term to interact strongly with the geometry. However, the main effect in 2d may just be that the fields are forced to lie in the neighborhood of zero field and as a consequence the field part of the action becomes irrelevant. Thus we can maybe expect a phase transition back to the $c \leq 1$ phase as a function of the mass. It might give us an unexpected easy way to study the phase transition related to the $c = 1$ barrier.

Acknowledgments. Zhang thanks J. Gizbert-Studnicki for help. This work is partly supported by the International PhD Projects Programme of the Foundation for Polish Science within the European Regional Development Fund of the European Union, agreement no. MPD/2009/6. JA thanks the Institute of Theoretical Physics and the Department of Physics and Astronomy at Utrecht University for hospitality and financial support. J.A. and A.G. acknowledge financial support by the Danish Research Council (FNU) from the grant "quantum gravity and the role of black holes".

References

[1] F. David, *Planar Diagrams, Two-Dimensional Lattice Gravity and Surface Models,* Nucl. Phys. **B257** (1985) 45.
A. Billoire and F. David, *Microcanonical simulations of randomly triangulated planar random surfaces*, Phys. Lett. B **168** (1986) 279-283.
V. A. Kazakov, A. A. Migdal, I. K. Kostov, *Critical Properties of Randomly Triangulated Planar Random Surfaces,* Phys. Lett. **B157** (1985) 295-300.
D.V. Boulatov, V.A. Kazakov, I.K. Kostov and A.A. Migdal, *Analytical and numerical study of the model of dynamically triangulated random surfaces*, Nucl. Phys. B **275** (1986) 641-686.
J. Ambjorn, B. Durhuus and J. Fröhlich, *Diseases of triangulated random surface models, and possible cures*, Nucl. Phys. B **257** (1985) 433-449;
J. Ambjorn, B. Durhuus, J. Fröhlich and P. Orland, *The appearance of critical dimensions in regulated string theories*, Nucl. Phys. B **270** (1986) 457-482.
J. Jurkiewicz, A. Krzywicki and B. Petersson, *A Numerical Study Of Dis-*

crete Euclidean Polyakov Surfaces, Phys. Lett. B **168** (1986) 273.
J. Jurkiewicz, A. Krzywicki and B. Petersson, *A Grand Canonical Ensemble Of Randomly Triangulated Surfaces,* Phys. Lett. B **177** (1986) 89.

[2] A. M. Polyakov, *Quantum Geometry of Bosonic Strings,* Phys. Lett. B **103** (1981) 207.

[3] J. Ambjorn and J. Jurkiewicz, *Four-dimensional simplicial quantum gravity*, Phys. Lett. B **278** (1992) 42-50.

M.E. Agishtein and A.A. Migdal, *Simulations of four-dimensional simplicial quantum gravity*, Mod. Phys. Lett. A **7** (1992) 1039-1062.
J. Ambjorn, S. Varsted, *Three-dimensional simplicial quantum gravity,* Nucl. Phys. **B373** (1992) 557-580.
J. Ambjorn, S. Varsted, *Entropy estimate in three-dimensional simplicial quantum gravity,* Phys. Lett. **B266** (1991) 285-290.
J. Ambjorn, D. V. Boulatov, A. Krzywicki, S. Varsted, *The Vacuum in three-dimensional simplicial quantum gravity,* Phys. Lett. **B276** (1992) 432-436.
M. E. Agishtein, A. A. Migdal, *Three-dimensional quantum gravity as dynamical triangulation,* Mod. Phys. Lett. **A6** (1991) 1863-1884.
D. V. Boulatov, A. Krzywicki, *On the phase diagram of three-dimensional simplicial quantum gravity,* Mod. Phys. Lett. **A6** (1991) 3005-3014.

[4] P. Bialas, Z. Burda, A. Krzywicki, B. Petersson, *Focusing on the fixed point of 4-D simplicial gravity,* Nucl. Phys. **B472** (1996) 293-308. [hep-lat/9601024].
S. Catterall, R. Renken, J. B. Kogut, *Singular structure in 4-D simplicial gravity,* Phys. Lett. **B416** (1998) 274-280. [hep-lat/9709007].

[5] S. Bilke, Z. Burda, A. Krzywicki, B. Petersson, J. Tabaczek and G. Thorleifsson, *4-D simplicial quantum gravity interacting with gauge matter fields,* Phys. Lett. B **418** (1998) 266; [hep-lat/9710077].
4-D simplicial quantum gravity: Matter fields and the corresponding effective action, Phys. Lett. B **432** (1998) 279 [hep-lat/9804011].
J. Ambjorn, K. N. Anagnostopoulos and J. Jurkiewicz, *Abelian gauge fields coupled to simplicial quantum gravity,* JHEP **9908** (1999) 016 [hep-lat/9907027].
S. Horata, H. S. Egawa, N. Tsuda, T. Yukawa, *Phase structure of four-dimensional simplicial quantum gravity with a U(1) gauge field,* Prog. Theor. Phys. **106** (2001) 1037-1050. [hep-lat/0004021].

[6] J. Ambjorn and R. Loll, *Nonperturbative Lorentzian quantum gravity, causality and topology change,* Nucl.Phys.B **536** (1998) 407-434, [hep-th/9805108].

[7] J. Ambjorn, J. Jurkiewicz and R. Loll: *Non-perturbative 3d Lorentzian quantum gravity,* Phys. Rev. D **64** (2001) 044011 [hep-th/0011276];

[8] J. Ambjorn, J. Jurkiewicz and R. Loll, *Dynamically triangulating Lorentzian quantum gravity,* Nucl.Phys.B **610** (2001), 347-382 [hep-th/0105267].
A non-perturbative Lorentzian path integral for gravity, Phys. Rev. Lett. **85** (2000) 924 [hep-th/0002050].

[9] S. Catterall, G. Thorleifsson, J. B. Kogut, R. Renken, *Singular vertices and the triangulation space of the D sphere,* Nucl. Phys. **B468** (1996) 263-276. [hep-lat/9512012].
P. Bialas, Z. Burda, B. Petersson and J. Tabaczek, *Appearance of mother universe and singular vertices in random geometries,* Nucl. Phys. B 495 (1997) 463-476 [hep-lat/9608030].

[10] P. Horava, *Quantum Gravity at a Lifshitz Point,* Phys. Rev. **D79** (2009) 084008. [arXiv:0901.3775 [hep-th]].
P. Horava, C. M. Melby-Thompson, *General Covariance in Quantum Gravity at a Lifshitz Point,* Phys. Rev. **D82** (2010) 064027. [arXiv:1007.2410 [hep-th]].
Anisotropic Conformal Infinity, [arXiv:0909.3841 [hep-th]].

[11] J. Ambjorn, J. Jurkiewicz and R. Loll, *Spectral dimension of the universe*, Phys. Rev. Lett. **95** (2005) 171301 [hep-th/0505113].

[12] P. Hořava: *Spectral dimension of the universe in quantum gravity at a Lifshitz point,* Phys. Rev. Lett. **102** (2009) 161301 [arXiv:0902.3657, hep-th].

[13] J. Ambjorn, A. Görlich, S. Jordan, J. Jurkiewicz and R. Loll, *CDT meets Horava-Lifshitz gravity*, Phys. Lett. B **690** (2010) 413-419 [arXiv:1002.3298, hep-th].

[14] C. Anderson, S. Carlip, J. H. Cooperman, P. Horava, R. Kommu and P. R. Zulkowski, *Quantizing Horava-Lifshitz Gravity via Causal Dynamical Triangulations,* arXiv:1111.6634 [hep-th].

[15] J. Ambjorn, J. Jurkiewicz and R. Loll, *Emergence of a 4D world from causal quantum gravity*, Phys. Rev. Lett. **93** (2004) 131301 [hep-th/0404156].
Reconstructing the universe, Phys. Rev. D **72** (2005) 064014 [hep-th/0505154].

[16] J. Ambjorn, A. Görlich, J. Jurkiewicz and R. Loll, *Planckian birth of the quantum de Sitter universe*, Phys. Rev. Lett. **100** (2008) 091304 [arXiv:0712.2485, hep-th].

The nonperturbative quantum de Sitter universe, Phys. Rev. D **78** (2008) 063544 [arXiv:0807.4481, hep-th].
Semiclassical universe from first principles, Phys. Lett. B **607** (2005) 205-213 [hep-th/0411152].
J. Ambjorn, A. Görlich, J. Jurkiewicz and R. Loll, *Geometry of the quantum universe*, Phys. Lett. B **690** (2010) 420-426 [arXiv:1001.4581, hep-th].

[17] J. Ambjorn, S. Jordan, J. Jurkiewicz and R. Loll, *A Second-order phase transition in CDT,* Phys. Rev. Lett. **107** (2011) 211303 [arXiv:1108.3932 [hep-th]].

[18] J. Ambjorn, J. Jurkiewicz, R. Loll and G. Vernizzi: *Lorentzian 3d gravity with wormholes via matrix models*, JHEP **0109** (2001) 022 [hep-th/0106082]; *3D Lorentzian quantum gravity from the asymmetric ABAB matrix model*, Acta Phys. Polon. B **34** (2003) 4667-4688 [hep-th/0311072];
J. Ambjorn, J. Jurkiewicz and R. Loll: *Renormalization of 3d quantum gravity from matrix models,* Phys. Lett. B **581** (2004) 255-262 [hep-th/0307263].
D. Benedetti, R. Loll and F. Zamponi: *(2+1)-dimensional quantum gravity as the continuum limit of causal dynamical triangulations*, Phys. Rev. D **76** (2007) 104022 [arXiv:0704.3214, hep-th].

[19] J. Laiho and D. Coumbe, *Evidence for Asymptotic Safety from Lattice Quantum Gravity,* Phys. Rev. Lett. **107** (2011) 161301 [arXiv:1104.5505 [hep-lat]].

[20] J. Ambjorn, J. Correia, C. Kristjansen, R. Loll, *On the relation between Euclidean and Lorentzian 2-D quantum gravity,* Phys. Lett. B **475** (2000) 24-32. [hep-th/9912267].

[21] J. Ambjorn, R. Loll, W. Westra and S. Zohren, *Putting a cap on causality violations in CDT,* JHEP **0712** (2007) 017 [arXiv:0709.2784, gr-qc];
A matrix model for 2D quantum gravity defined by causal dynamical triangulations, Phys. Lett. B **665** (2008) 252-256 [arXiv:0804.0252, hep-th];
A new continuum limit of matrix models, Phys. Lett. B **670** (2008) 224 [arXiv:0810.2408, hep-th].
J. Ambjorn, R. Loll, Y. Watabiki, W. Westra and S. Zohren: *A string field theory based on causal dynamical triangulations,* JHEP **0805** (2008) 032 [arXiv:0802.0719, hep-th].

[22] H. Kawai, N. Kawamoto, T. Mogami and Y. Watabiki, *Transfer matrix formalism for two-dimensional quantum gravity and fractal structures of spacetime,* Phys. Lett. B **306** (1993) 19 [hep-th/9302133].

[23] J. Ambjorn and Y. Watabiki, *Scaling in quantum gravity,* Nucl. Phys. B **445** (1995) 129 [hep-th/9501049].

[24] J. Ambjorn, J. Jurkiewicz and Y. Watabiki, *On the fractal structure of two-dimensional quantum gravity,* Nucl. Phys. B **454** (1995) 313-342 [hep-lat/9507014].

[25] S. Catterall, G. Thorleifsson, M. J. Bowick, V. John, *Scaling and the fractal geometry of two-dimensional quantum gravity,* Phys. Lett. **B354** (1995) 58-68. [hep-lat/9504009].

[26] H. Aoki, H. Kawai, J. Nishimura and A. Tsuchiya, *Operator product expansion in two-dimensional quantum gravity,* Nucl. Phys. B **474** (1996) 512 [hep-th/9511117].

[27] V. G. Knizhnik, A. M. Polyakov and A. B. Zamolodchikov, *Fractal Structure of 2D Quantum Gravity,* Mod. Phys. Lett. A **3** (1988) 819.
F. David, *Conformal Field Theories Coupled to 2D Gravity in the Conformal Gauge,* Mod. Phys. Lett. A **3** (1988) 1651.
J. Distler and H. Kawai, *Conformal Field Theory and 2D Quantum Gravity Or Who's Afraid of Joseph Liouville?,* Nucl. Phys. B **321** (1989) 509.

[28] B. Durhuus, J. Frohlich and T. Jonsson, *Critical Behavior in a Model of Planar Random Surfaces,* Nucl. Phys. B **240** (1984) 453; Phys. Lett. B **137** (1984) 93.

[29] J. Ambjorn and B. Durhuus, *Regularized Bosonic Strings Need Extrinsic Curvature,* Phys. Lett. B **188** (1987) 253.

[30] J. Ambjorn, K.N. Anagnostopoulos and R. Loll: *A new perspective on matter coupling in 2d quantum gravity*, Phys. Rev. D **60** (1999) 104035 [hep-th/9904012].

[31] J. Ambjorn, K.N. Anagnostopoulos, R. Loll and I. Pushkina: *Shaken, but not stirred - Potts model coupled to quantum gravity,* Nucl. Phys. B **807** (2009) 251 [arXiv:0806.3506, hep-lat].

[32] S. Jain and S. D. Mathur, *World sheet geometry and baby universes in 2-D quantum gravity,* Phys. Lett. B **286** (1992) 239 [hep-th/9204017].

[33] J. Ambjorn, S. Jain, G. Thorleifsson, *Baby universes in 2-d quantum gravity,* Phys. Lett. **B307** (1993) 34-39. [hep-th/9303149].

[34] J. Ambjorn, B. Durhuus and T. Jonsson, *A Solvable 2-d gravity model with* $\gamma > 0$*,* Mod. Phys. Lett. A **9** (1994) 1221 [hep-th/9401137].

[35] M.E. Cates, Europhysics Lett. B **719** (1988).
A. Krzywicki, *On The Stability Of Random Surfaces,* Phys. Rev. D **41** (1990) 3086.

[36] P. Di Francesco, E. Guitter and C. Kristjansen, *Integrable 2-D Lorentzian gravity and random walks,* Nucl. Phys. B **567** (2000) 515 [hep-th/9907084].

[37] M.E.J Newman and T. Barkama, *Monte Carlo Methods in Statistical Physics*, Oxford University Press 2002.

[38] J. Ambjorn, B. Durhuus, T. Jonsson and G. Thorleifsson, *Matter fields with* $c > 1$ *coupled to 2-d gravity,* Nucl. Phys. B **398** (1993) 568 [hep-th/9208030].

[39] F. David, *A Scenario for the* $c > 1$ *barrier in noncritical bosonic strings,* Nucl. Phys. B **487** (1997) 633 [hep-th/9610037].

[40] D. Benedetti and J. Henson, *Spectral geometry as a probe of quantum spacetime,* Phys. Rev. D **80** (2009) 124036 [arXiv:0911.0401 [hep-th]].

[41] J. Ambjorn, P. de Forcrand, F. Koukiou and D. Petritis, *Monte Carlo Simulations Of Regularized Bosonic Strings,* Phys. Lett. B **197** (1987) 548.

[42] O. Lauscher and M. Reuter, *Fractal spacetime structure in asymptotically safe gravity,* JHEP **0510** (2005) 050 [hep-th/0508202].